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Contracting for information: On the effects of the principal's outside option

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Abstract

We study optimal contracting for information in a setting where an uninformed principal has the opportunity to undertake an outside option. The contract specifies a decision rule and a transfer for each unit of information revealed by the agent. Due to the existence of the outside option, the informational rent is nonmonotonic, and we characterize the properties of the optimal contract. We show that the outside option represents an important pressure for the agent because it allows the principal to punish him severely with negative transfers. Moreover, we compare our optimal contract to the one under perfect commitment without an outside option developed by Krishna and Morgan [2008]. We find that regardless of the divergence of preferences between the principal and the agent, the contract with an outside option is always better for the principal. Furthermore, we show that the use of an outside option increases information extraction.

Keywords: Contract, Information, Outside option, Transfers

JEL Classification: D23; D82.

1 Introduction

From a strategic management approach, one basic principle of decision making is to decentralize authority to those who have information (see, for instance, Milgrom and Roberts [1992] or Saloner et al. [2001]), but it is rarely optimal to do so. As a result, a decision or a project is rarely implemented by the agent who has information. Indeed, because of vertical relationships in firms, even if subordinates are better informed than managers, they rarely have the power to make decisions. Consequently, the authority given to subordinates is not absolute, and they are generally consulted by managers for their private information. In addition, subordinates may

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not adhere to the manager's vision for the firm; thus, conflicting preferences between agents may make the delegation of authority unfavorable to managers[1].

Moreover, even if a manager does not usually have all of the relevant information, he possesses some ability to make decisions. More precisely, he can commit himself to what we call an *outside option,* that is, a decision he can make without the use of a subordinate's private information. Indeed, when a subordinate is not cooperative, there are situations in which it is not possible to do nothing, and the manager must make a decision. For example, consider a manager who must renew the hardware of his subordinate, and imagine that he is not fully aware of the subordinate's needs. We can imagine that, either the manager directly asks the subordinate to communicate his needs, either he makes a decision based on what he thinks will be good for the subordinate. In such a case, the manager has an outside option that he can pursue in the absence of coordination with his subordinate. However, it is possible that the outside option is unfavorable to the subordinate and the manager can use it as a pressure to optimally extract private information from him. Thus, whereas intuition suggests that managers should always employ people whose biases are as low as possible, the use of an outside option may radically change the result.

In this paper, we investigate the effects of such an outside option in a principal-agent setting in which the principal has the ability to perfectly commit to a contract that specifies a decision rule and a monetary transfer for each unit of information revealed by the agent. We consider a standard principal-agent model in which the principal is considered as a Stackelberg leader. By definition, the outside option does not depend on the information communicated by the agent. Preferences are misaligned, and the agent is characterized by a bias that is perfectly observed by the principal. As in Krishna and Morgan [2008], we enrich the Crawford-Sobel [1982] model by allowing for the possibility of contractual monetary transfers. Moreover, as in the previous example, we assume that the principal has the opportunity to undertake an outside option which directly affects the reservation utility of the agent. Thus without interaction the agent's payoff will be determined by the outside option chosen by the principal. Consequently, the reservation utility of the agent depends on the principal's outside option and his private information. Due to these hypotheses, the informational rent is nonmonotonic and may vanish for some states of nature (Jullien [2000]).

This study focuses on a mechanism by which the principal, according to an optimal contract, chooses an action and a monetary transfer that are payoff-relevant for both agents. Our goal is to study how the structure of an optimal contract is affected by the use of an outside option by the principal. We first present some general properties of the optimal contract. Next, we use the so-called uniform-quadratic case to explicitly characterize optimal contracts according to several hypotheses concerning the principal's behavior. To determine the value of the outside option, we consider two types of behaviors of the principal. In the first type, we assume that the principal is *naive* and that the outside option is only based on the principal's prior belief in the

[1] Following the work of Holmström [1977/1984], some papers have studied the optimal way to fully delegate decision making to better-informed agents. For instance, Dessein [2002] shows that under some hypotheses on preferences, delegation can dominate cheap-talk communication.

state of nature. In the second one, we consider a more *sophisticated* principal that determines an outside option on the basis of the bias of the agent. We show that a sophisticated principal can select an outside option that generates a higher expected payoff than the one of a naive principal.

In a general setting, our results show that the design of the optimal contract depends on the evolution of the informational rent. We characterize a partition contract with at most three separate pieces. For interior states of nature, the optimal contract specifies an action equal to the outside option. For other states of nature, an optimal contract involves decisions that are responsive to the state and may induce negative transfers. With these transfers, the principal pulls out some surplus from the agent due to the existence of the outside option. Negative transfers arise when the relative gap between the ideal action for the agent and the outside option is strong enough. In particular, whatever the divergence of preferences between the principal and the agent, Proposition 3 shows that for high states, transfers are always negative, whereas this is not always the case for low states. Furthermore, by using the uniform-quadratic case, we compare our contract with the one by Krishna and Morgan [2008], who study a similar problem without an outside option. Regardless of the divergence of preferences between the principal and the agent, we find that the contract with an outside option is always better from the principal's point of view; that is, the payoff to the principal is always superior in our case. Moreover, we show that the opportunity of using an outside option also increases information extraction. We consider that information extraction is due to responsive mechanisms, that is, optimal mechanisms in which the principal chooses a different action for each state. We conclude that our contract is better from the perspective of information extraction for a low value of the agent's bias. We also show that the higher the agent's bias is, the more the pressure induces by the outside option increases information revelation in our model.

Related literature. This paper contributes to the literature on contract theory when agents have type-dependent reservation utility (Lewis and Sappington, [1989*a*], [1989*b*]); Maggi and Rodriguez, [1995]; Jullien, [2000]; Rasul and Sonderegger, [2010])). It also contributes to the literature on strategic information transmission between an agent and a principal. Starting from the classic *cheap talk* model of Crawford and Sobel [1982], some papers suppose that the principal has commitment power (Melumad and Shibano, [1991]; Baron,[2000]; Ottaviani [2000]; Krishna and Morgan, [2008]; Ambrus and Egorov, [2012]). Chiba and Leong [2011] study the impact of adding an outside option in the cheap-talk setting of Crawford and Sobel [1982]. However, they define inaction as an outside option and consider a constant reservation utility. We contribute to this literature by introducing a type-dependent reservation utility based on an outside option that the principal can use as a pressure to control the agent.

In our model, the outside option is based on either the principal's prior information or the agent's bias. Consequently, the agent becomes inextricably involved with the project even if the principal does not interact with him. This model differs from the paper by Che, Dessein and Kartik [2013], in which the outside option is equivalent to the cancellation of a project. This model may also differ from Barron [2000], where the outside option is interpreted to be the status quo.

The remainder of the paper is organized as follows. In section 2, we present the principal-agent model and hypothesis with regard to the agent's individual rationality constraint. Section 3 presents some properties of the optimal contract. Section 4 characterizes the optimal contract in the uniform-quadratic case. By comparing our contract with the one developed in a perfect commitment setting by Krishna and Morgan [2008], we discuss the efficiency consequences of the outside option concerning information and surplus extraction. All proofs are available in the Appendix.

2 Model

2.1 Preliminaries

Consider a model of contracting for information between two agents $i \in \{A; P\}$: an agent (A) and an uninformed principal (P). The principal has authority to choose an action $y \in \mathbb{R}$ under uncertainty. The state of nature $\theta \in [0, 1]$ is distributed according to the cumulative distribution $F(.)$, which has an atomless and everywhere positive density $f = F'$. We assume that $f(.)$ is continuous and everywhere differentiable. The principal has no information about θ and refers to an agent that perfectly observes θ without cost.

We assume that the payoff functions of the agents are of the form $U(y, \theta, b_i)$ where $b_i > 0$ is a common-knowledge bias parameter that measures the conflict of interest among the two agents. The function $U(y, \theta, b_i)$ is twice-continuously differentiable and presents the usual characteristics of the derivative: $U_{11} < 0$, $U_{12} > 0$, $U_{13} > 0$ and $U_{122} \leq 0$[2].

The bias of the principal is normalized to be $b_P = 0$, whereas the agent's bias is positive and defined such that $b_A = b > 0$. Because of the bias, the two agents are in conflict in the action the principal must make. The most preferred action of the principal is to choose an action such that $y^P(\theta) = \arg\max U(y, \theta)$, and the most preferred action of the agent is $y^A(\theta) = \arg\max U(y, \theta, b)$. Because $U_{13} > 0$, $b > 0$ implies that $y^A(\theta) > y^P(\theta)$. In other words, the most preferred action of the agent always over-estimates the most preferred action of the principal.

We assume that the principal can use monetary transfers to extract information observed by the agent. For convenience of analysis, we suppose that the preferences of the two parties are quasi-linear with transfer. Thus, if a payment t is made to the agent, then the payoff (or individual surplus) of the principal from action y in state θ is

$$U(y, \theta, 0) - t$$

while the payoff (or individual surplus) of the agent is

$$U(y, \theta, b) + t$$

To model the contract for information between the principal and the agent, we follow a standard mechanism design approach. The principal can precommit himself to a non-renegotiable contract that stipulates which decision $y(.)$ and transfer $t(.)$ should be made as a function of

[2] See Fudenberg and Tirole (1991) [.], p.263.

the agent's report on the state of nature. From the Revelation Principle, there is no loss of generality in restricting the principal to offer a direct revelation mechanism $\{y(\widehat{\theta}), t(\widehat{\theta})\}$ with $\widehat{\theta} \in [0,1]$. Standard arguments show that under perfect commitment, necessary and sufficient conditions for incentive compatibility (IC) require that (i) $y(.)$ is nondecreasing and (ii) $t'(\theta) = -U_1(y,\theta,b)y'(\theta)$ at all points θ where $y(.)$ is differentiable (see, for instance, Salanié, [1997]).

2.2 The individual rationality constraint

The principal proposes a contract $\{y, t\}$ to the agent on *a take-it-or-leave-it basis* where the verifiable variables are the action y and the transfer t. Additionally, we suppose that an outside option, namely, $\widetilde{y}$, is available to the principal. The outside option only depends on prior information available for the principal that is prior distribution on θ and the perfect observation of the bias parameter b. Due to these informations, the determination of the outside option can traduce several behaviors of the principal. We will discuss the principal's behaviors in the uniform-quadratic model. The results presented in the general case are independent of such a hypothesis on the behavior of the principal.

The reservation utility of the agent $U(\widetilde{y},\theta,b)$ depends on the outside option, his private information and his bias. We consider the situation where the agent may opt out of the contract after observing θ. In this case, the individual rationality constraint has to be satisfied state by state such that

$$\begin{aligned} U(y,\theta,b) + t(\theta) &\geq U(\widetilde{y},\theta,b) \\ t(\theta) &\geq U(\widetilde{y},\theta,b) - U(y,\theta,b) \end{aligned}$$

for all $\theta \in [0,1]$. The transfer must be given incentives to the agent to report the truth given the outside option. From this constraint, when $U(y,\theta,b) > U(\widetilde{y},\theta,b)$ the agent strictly prefers to reveal information and is even willing to pay for it (negative transfers).

This constraint can be rewritten as follows

$$U^A(\theta) = U(y,\theta,b) + t(\theta) - U(\widetilde{y},\theta,b) \geq 0 \qquad (IR)$$

where $U^A(\theta)$ can be interpreted as the informational rent of the agent. The evolution of the marginal rent is decisive to characterize the properties of the optimal contracts, as we show below. Due to the incentive compatibility constraint, the evolution of the marginal rent is defined by

$$\frac{dU^A(\theta)}{d\theta} = U_2(y,\theta,b) - U_2(\widetilde{y},\theta,b).$$

Because $U_{12} > 0$, we must consider three cases:

- (a) if $y(\theta) < \widetilde{y}$ then $\frac{dU^A(\theta)}{d\theta} < 0$
- (b) if $y(\theta) = \widetilde{y}$ then $\frac{dU^A(\theta)}{d\theta} = 0$
- (c) if $y(\theta) > \widetilde{y}$ then $\frac{dU^A(\theta)}{d\theta} > 0$.

In such a case, informational rent is nonmonotonic and may vanish for intermediate values of θ (Jullien, [2000]). The monotonicity of the informational rent depends on the relative position between the action chosen by the principal $y(\theta)$ and her outside option $\widetilde{y}$. If $y(\theta) = \widetilde{y}$, the principal's action is to undertake his outside option $\widetilde{y}$, and the agent obtains his reservation utility level $U(\widetilde{y}, \theta, b)$. Consequently, to minimize the cost generated by the transfer, the agent receives no transfer. If $y(\theta) \neq \widetilde{y}$, the gain for the agent to reveal information increases with the difference between $y(\theta)$ and $\widetilde{y}$. Figure (1) describes the evolution of the informational rent in the case there exists θ_1, θ_2 such that $0 < \theta_1 < \theta_2 < 1$.

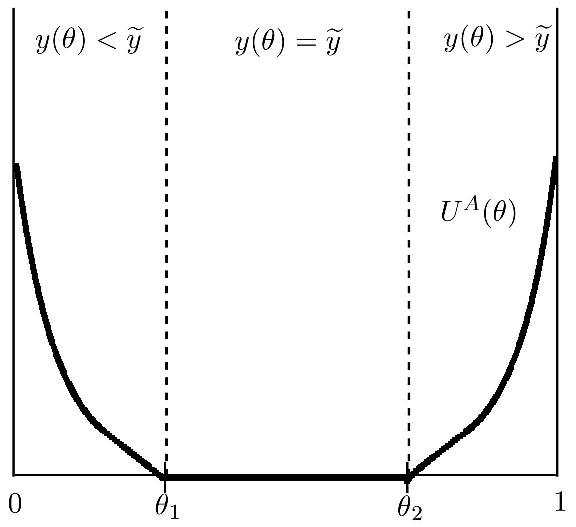

Figure (1): Evolution of the informational rent $U^A(\theta)$

Remarks: *a*) The evolution of the informational rent involves a partition contract with at most three separate pieces. In the next section, we will present a sufficient condition for the existence of a subinterval $[\theta_1, \theta_2]$ in the general case. Furthermore, the uniform-quadratic case indicates that the steps θ_1 and θ_2 may be parameterized by the value of the bias b. *b*) In addition, according to the individual rationality constraint, $t(\theta) \geq U(\widetilde{y}, \theta, b) - U(y, \theta, b)$ can be either positive or negative. As in Baron [2000], we assume that negative transfers from the principal to the agent are feasible. In this case, we suppose that no limited liability clause exists to protect the agent, and his individual rationality only involves $U^A(\theta) \geq 0$.

In the next section, we derive some qualitative features and properties of the optimal contract with a transfer and an outside option.

3 Properties of the optimal contract

3.1 The control problem

From the definition of the informational rent, we have $t(\theta) = U^A(\theta) + U(\widetilde{y}, \theta, b) - U(y, \theta, b)$. By substituting the transfer $t(\theta)$ in the principal's objective and denoting $\Phi(y, \theta, b) = U(y, \theta) + U(y, \theta, b) - U(\widetilde{y}, \theta, b)$, the optimal contract becomes the solution to the following control problem

(P) :

$$\max_{y(\theta)} \int_0^1 \left[\Phi(y,\theta,b) - U^A(\theta)\right] dF(\theta)$$

subject to the law of motion

$$\frac{dU^A(\theta)}{d\theta} = U_2(y,\theta,b) - U_2(\widetilde{y},\theta,b)$$

and the constraint

$$U^A(\theta) \geq 0,$$

where $U^A(\theta)$ is the state variable and y is the control variable.

The Hamiltonian of this problem is

$$H = \left[\Phi(y,\theta,b) - U^A(\theta)\right] f(\theta) + \mu\left[U_2(y,\theta,b) - U_2(\widetilde{y},\theta,b)\right],$$

where μ is the costate variable. Now, we write the associated Lagrangien

$$L = \left[\Phi(y,\theta,b) - U^A(\theta)\right] f(\theta) + \mu\left[U_2(y,\theta,b) - U_2(\widetilde{y},\theta,b)\right] + \lambda U^A(\theta),$$

where λ is the multiplier associated with the state variable $U^A(\theta)$.

We solve (P) by using standard results of optimal control problems (see for example, Chiang, [1991]). The first-order condition for the maximization of the Hamiltonian with respect to y implies that

$$\frac{\partial H}{\partial y} = 0 \Rightarrow \Phi_1(y,\theta,b)f(\theta) + \mu U_{12}(y,\theta,b) = 0. \tag{1}$$

If $\Phi_{11}(y,\theta,b)f(\theta) + \mu U_{112}(y,\theta,b) < 0$, condition (1) is sufficient for this maximization[3]. The other sufficient conditions are

$$-\frac{\partial L}{\partial U^A} = \frac{d\mu}{d\theta} = f(\theta) - \lambda \tag{2}$$

$$\frac{dU^A(\theta)}{d\theta} = U_2(y,\theta,b) - U_2(\widetilde{y},\theta,b) \tag{3}$$

$$\lambda(\theta)\, U^A(\theta) = 0,\ \lambda(\theta) \geq 0,\ U^A(\theta) \geq 0 \tag{4}$$

$$\mu(0) = 0 \text{ and } \mu(1) = 0. \tag{5}$$

The transversality conditions given in condition (5) are determined by applying the theory of optimal control (Chiang, [1991]). They are consistent with the case described in Figure (1) where the state variable is free at the two extremes.

[3] In the quadratic loss case, $U_{112} = 0$ so $U_{11}(y,\theta,b)$ is enough for concavity.

3.2 The general results

Proposition 1 exhibits a direct result of solving the problem (P) concerning the most-preferred action of the two agents.

Proposition 1. *$y^A(\theta)$ and $y^P(\theta)$ are not implementable in an optimal contract.*

Proposition 1 simply notes that the ideal actions for the principal and the agent are never implementable in the optimal contract. Krishna and Morgan [2008] show that in the uniform-quadratic case, for low bias, delegation $\left(\text{that is } y(\theta) = y^A(\theta)\right)$ represents a piece of his optimal contract for states between b and $1-2b$. We will see in the next section that due to the existence of the outside option, such a result is never optimal in our setting. Ottaviani [2000] presents the transfer necessary to fully align the preferences of the agent to those of the principal and thus to implement $y(\theta) = y^P(\theta)$. However, he does not study the optimal contract.

To solve the problem (P), we first establish that the form of optimal action $y^*(\theta)$ depends on the monotonicity of the informational rent, and we determine all of the possible cases for $\mu(\theta)$ according to conditions (2) to (5). Then, we have the following results:

Lemma 1. *The optimal scheme associated with the costate variable $\mu^*(\theta)$ is such that*

$$\mu^*(\theta) = \begin{cases} F(\theta) \text{ if } y(\theta) < \widetilde{y} \\ F(\theta) - \int_{\theta_1}^{\theta} \lambda(s)\, ds \text{ if } y(\theta) = \widetilde{y} \text{ and for } \theta_1 < 1. \\ F(\theta) - 1 \text{ if } y(\theta) > \widetilde{y} \end{cases}$$

$\mu(\theta)$ represents the multiplier associated with the evolution of the marginal informational rent. Because $U_{122} \leq 0$, if $y(\theta) < \widetilde{y}$, then $\frac{d^2U^A(\theta)}{d\theta^2} > 0$, and if $y(\theta) > \widetilde{y}$, then $\frac{d^2U^A(\theta)}{d\theta^2} < 0$, which is consistent with Lemma 1.

To prove the existence of a subintervall $[\theta_1, \theta_2]$ where $U^A(\theta) = 0$ and $y(\theta) = \widetilde{y}$, we have to verify that $\lambda(\theta) > 0$ because Lemma 2 implies that $\mu(\theta_1) \neq \mu(\theta_2)$. Condition (2) implies that $\lambda(\theta) = f(\theta) - \mu'(\theta)$. The next lemma establishes a sufficient condition such that the principal can take its outside option as an optimal action. As a consequence, an optimal contract can involve some pooling of intermediary states. Thus, even though the principal can implement full revelation, this is too expensive and never optimal on the subinterval $[\theta_1, \theta_2]$.

Lemma 2. *A sufficient condition for $\lambda(\theta) > 0$ is given by*

$$\Phi_1(\widetilde{y}, \theta, b)\left[f'(\theta)\, U_{12}(\widetilde{y}, \theta, b) - U_{122}(\widetilde{y}, \theta, b) f(\theta)\right] \geq 0.$$

We note that this sufficient condition is always satisfied in the uniform-quadratic case. Now, we characterize some proporties of $y^*(\theta)$ that satisfy all of the optimal conditions, including condition (1). Define $y^*(\theta) = y^H(\theta)$ (respectively, $y^*(\theta) = y^L(\theta)$) the optimal action for $\mu^*(\theta) = F(\theta)$ (respectively, $\mu^*(\theta) = F(\theta) - 1$). Proposition 2 states the results of the comparison across the optimal action $y^*(\theta)$ following the value of the state of nature.

Proposition 2. *An optimal action $y^*(\theta)$ has the following features:*

1. $\forall\theta \in [0, \theta_1]$, *i)* $y^*(\theta) = y^H(\theta) > y^P(\theta)$ *and ii)* $y^A(\theta) > y^H(\theta) > y^P(\theta)$ *if* $t(\theta)$ *is nonincreasing;*

2. $\forall\theta \in [\theta_1, \theta_2]$, $y^*(\theta) = \widetilde{y}$;
3. $\forall\theta \in [\theta_2, 1]$, $y^*(\theta) = y^L(\theta) < y^A(\theta)$;
4. $\forall\theta \in [0, 1]$, $y^H(\theta) > y^L(\theta)$.

Some properties on the optimal action $y^*(\theta)$ can be derived from the previous proposition. For low and high states of nature, the action chosen is always responsive to the state of nature; thus, a transfer is implemented between the principal and the agent. Moreover, for low states and a nonincreasing transfer, the optimal action is lower than delegation. This is always the case for high states. The next proposition shows that the optimal transfer can be negative.

Proposition 3. For any $\theta \in (\theta_2, 1]$, $t(\theta) < 0$. Moreover, there exists $\underline{b}, \overline{b}$ such that if $b \in (\underline{b}, \overline{b})$ $t(\theta) < 0$ for all $\theta \in [0, \theta_1)$.

Proposition 3 implies that negative transfers are feasible in the optimal contract for some states. Because the interests of the two parties are divergent, a positive transfer is usually used to induce the agent to reveal his private information. However, for some states of nature and some values of the agent's bias, the outside option reduces the value of the transfer to make it negative. For high states, the transfer is always negative, whereas for low states, its sign depends on the distortion of preferences between the two agents. By implementing a negative transfer, the principal captures some individual surplus from the agent due to the existence of the outside option if the agent does not participate in the contract. Because the ideal action for the agent always over-estimates the ideal action of the principal, the higher the realized state, the more unfavorable the outside option becomes for the agent. However, in low states and for preferences that are sufficiently divergent ($b > \overline{b}$), the principal implements a positive transfer because the difference between the ideal action for the agent and the outside option is small.

In the next section, we propose an explicit characterization of the optimal contract for the uniform-quadratic case, as defined in Appendix 2. Starting from Crawford and Sobel [1982], the uniform-quadratic case is extensively used by the literature to compare mechanisms in which the decision-maker (principal) and expert (agent) interact.

4 The optimal contract in the uniform-quadratic case

Concretely, we consider that θ is distributed uniformly on $[0, 1]$ and restrict payoff functions to the quadratic loss case

$$\begin{aligned} U(y, \theta) &= -(y - \theta)^2 \\ U(y, \theta, b) &= -(y - \theta - b)^2 \end{aligned}$$

where $b > 0$.

This section helps us clearly explain the characteristics of the optimal contract based on the bias of the agent. Therefore, we compare several contracts according to two criteria: a) the revelation of information and b) the extraction of surplus.

First, we consider a benchmark case in which the reservation utility of the agent is equal to zero. This case helps us understand how the outside option allows the principal to extract surplus from the agent. Second, we suppose that an outside option exists for the principal. We retain two types of outside options. The first is based on a naive behavior of the principal, whereas the second describes a more sophisticated behavior. In the naive behavior, the principal does not account for the bias of the agent, and we suppose that the outside option only depends on the principal's prior belief about the state of nature. In the sophisticated behavior, we assume that the principal has the ability to determine an outside option depending on the bias of the agent. To do so, we proceed in two steps. In a first step, we consider a feasible decision rule according to conditions (7) to (12) established in Appendix 2. This feasible rule depends on a parameter k, which defines the values of $\theta_1(k)$ and $\theta_2(k)$. In a second step, we optimally determine the value of k by maximizing the principal's expected payoff. We find that the outside option explicitly depends on the bias of the agent.

We conclude the paper by comparing the efficiency of our contracts with those developed in the seminal paper of Krishna and Morgan [2008]. Efficiency is compared according to the extraction of surplus and information revelation.

4.1 The benchmark case

We begin by examining a somewhat standard problem in which the utility reservation of the agent is normalized to zero. In such a case, the principal has no opportunity to take an outside option, which implies that

$$-(\widetilde{y} - \theta - b)^2 = 0.$$

Notice that for the particular case of quadratic loss functions, this situation is equivalent to the one in which the principal appoints the agent to take $y^A = \theta + b$, that is, delegation. The individual rationality constraint becomes

$$t(\theta) \geq (y - \theta - b)^2 \geq 0 \tag{6}$$

Without an outside option, because $t(\theta) \geq (y - \theta - b)^2 \geq 0$, only nonnegative transfers from the principal to the agent are feasible. In effect, the agent is protected by a sort of "limited liability" clause and cannot be punished too severely. Consequently, the principal cannot extract surplus from the agent. Hence, the revelation of information is costly to the principal because he must pay the agent to extract his private information.

Krishna and Morgan [2008] consider a similar setup but restrict their attention to the case where $t(\theta) \geq 0$. Their optimal contract does not satisfy condition (6), which is more binding. In effect, they find that there exists an interval in which the action is unresponsive to the state, that is, $\overline{y} = a$ and $t = 0$ and this never satisfies condition (6). The unique action that does not induce transfers and meets our individual rationality constraint is the delegation. The next proposition presents this optimal contract in such a case.

Proposition 4. *The optimal contract* $(y(\theta), t(\theta))$ *without an outside option is*

$$\begin{cases} y(\theta) = \frac{3}{2}\theta + \frac{b}{2};\ t(\theta) = \frac{3}{4}(\theta - b)^2\ \ \forall\theta \in [0, b] \\ y(\theta) = \theta + b;\ t(\theta) = 0\ \forall\theta \in [b, 1] \end{cases}$$

The optimal contract is defined for $b < 1$ and consists of only two pieces, as described in figure (2). In low states, the action y remains between the ideal action for the principal and the ideal action for the agent. In high states, the action that is the best for the agent $(y(\theta) = \theta + b)$ is chosen, and no transfers are made. The more the agent is biased, that is, the more b increases, the less delegation becomes optimal. The payoff to the principal under this contract is $-b^2 + \frac{1}{6}b^3$. Note that the payoff of the principal decreases with the agent's bias.

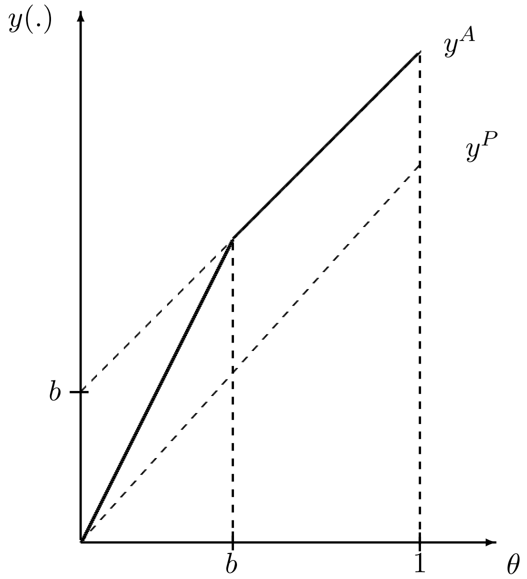

Figure (2): Optimal action $y(.)$ without an outside option

By comparing these results with those obtained by Krishna and Morgan [2008], because the individual rationality constraint (6) is more binding than the one used in their model, it is obvious that their contract is better with regard to the principal's expected payoff. However, the contract developed in the benchmark case is better with regard to information extraction because it is never unresponsive to the state.

The aim of the benchmark case is not to compare our results with those of the related literature. Indeed, this case is useful to provide some intuitions concerning the rent extraction and to introduce the way in which the principal can use the threat of the outside option to extract surplus from the agent. Without an outside option, we have shown that only nonnegative transfers are implementable. Thus, in our model, only the presence of the outside option allows the principal to compel the agent's payoff credibly and hence to increase the principal's ex-ante expected payoff, as we show. Notice that in our model the principal does not strategically choose the outside option. Indeed, we assume that she never depends on the agent's behavior. On the contrary, we examine two kinds of outside options that depend on the principal's behavior.

4.2 The naive principal

We first consider a naive behavior in which the principal basically selects the outside option according to his prior belief about the state of nature. We have $\widetilde{y} = \arg\max \int_0^1 -(y-\theta)^2 d\theta = \frac{1}{2}$.

The uniform-quadratic program is solved in Appendix 2. In the uniform-quadratic case, the relevant convexity conditions are satisfied; thus, conditions (7) to (12) are also sufficient. In such a case, if $b < 1$, an optimal contract is divided into three separate pieces. In low states, that is, $\forall\theta \in [0, \frac{1-b}{3}]$, the optimal action $y^H(\theta) = \frac{3}{2}\theta + \frac{b}{2}$ is chosen, and the associated transfer is $t^H(\theta) = \frac{3}{4}\left[(\theta - b)^2 - \frac{1}{9}(1-4b)^2\right]$. As θ increases, the associated transfer in absolute terms decreases and becomes zero for θ between $\theta_1 = \frac{1-b}{3}$ and $\theta_2 = \frac{2-b}{3}$. Thus, for such values of θ, the optimal action is to choose the principal's outside option: $\widetilde{y} = \frac{1}{2}$. For high states, that is, $\forall\theta \in [\frac{2-b}{3}, 1]$, the optimal action $y^L(\theta) = \frac{3}{2}\theta + \frac{b-1}{2}$ is chosen, and the associated negative transfer is $t^L(\theta) = \frac{3}{4}\left[(\theta - b - 1)^2 - \frac{1}{9}(1+4b)^2\right]$. Figure (3) represents the optimal contract $(y(.), t(.))$ for $b \in [\frac{1}{7}; \frac{1}{4}]$. We observe that for $\theta \in [0, \theta_1]$, the optimal transfer becomes negative for values of θ close to θ_1. Figure (4) represents the other cases concerning the transfer. Note that the optimal scheme for $y^*(\theta)$ is the one shown in Figure (3). More generally, we have the following results concerning the sign of the transfer.

Proposition 5. *i)* $\forall\theta \in [0, \frac{1-b}{3}]$, $t^H(\theta) \leq 0$ *if* $b \leq \frac{1}{7}$ *and* $t^H(\theta) \geq 0$ *if* $\frac{1}{4} \leq b < 1$.
ii) $\forall\theta \in]\frac{2-b}{3}, 1]$, $t^L(\theta) < 0$.

The qualitative features of the transfer when the bias is low differ somewhat from the case in which the bias is high. In particular, we observe that $t(\theta) \leq 0$ if $b \leq \frac{1}{7}$ and $t(\theta) \geq 0$ if $\frac{1}{4} \leq b < 1$ for $\theta \in [0, \theta_1]$. Thus, we can conclude that the lower the bias of the agent, the more the principal can capture some individual surplus from the agent with a negative transfer.

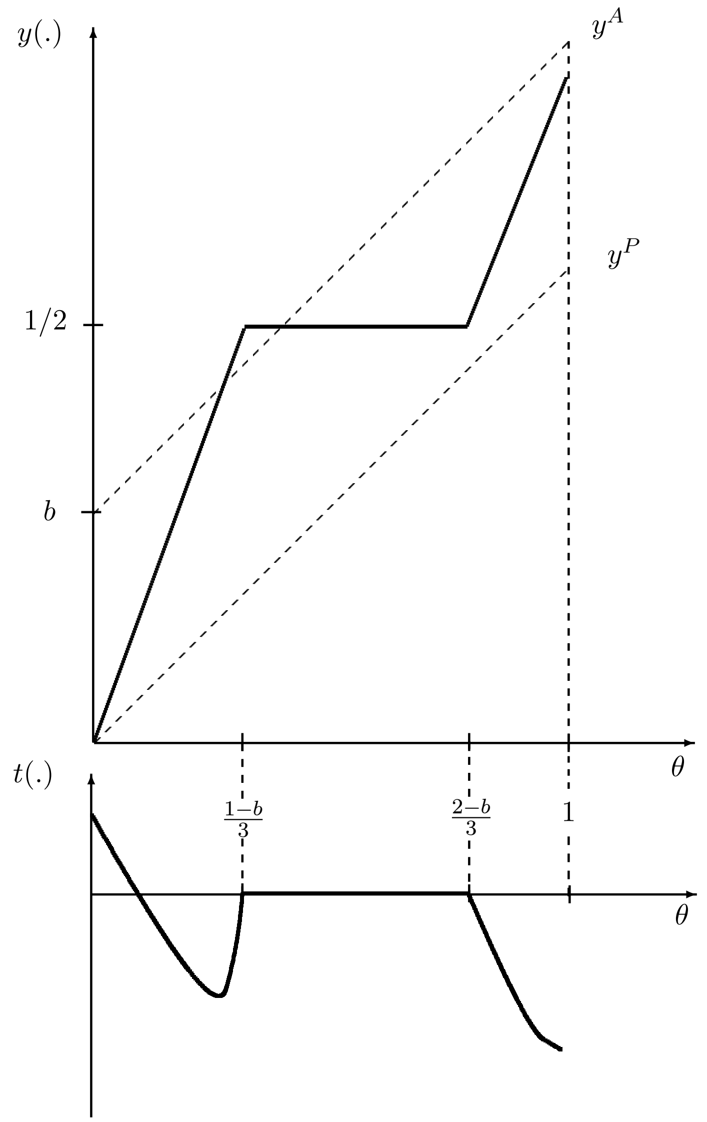

Figure (3): Optimal contract $(y(.), t(.))$ for a naive principal and for $b \in [\frac{1}{7}, \frac{1}{4}]$

Figure (4) represents the other cases for the optimal transfers according to the other values of the agent's bias.

Optimal transfer if $b \leq \frac{1}{7}$

Optimal transfer if $\frac{1}{4} \leq b < 1$

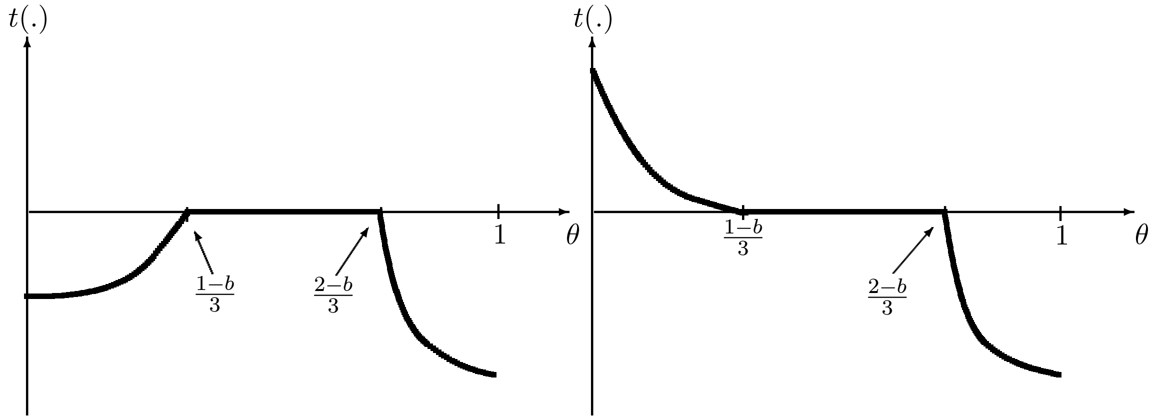

Figure (4): Optimal transfer $t(.)$ for a naive principal and for $b \leq \frac{1}{7}$ and $\frac{1}{4} \leq b < 1$

The outside option is optimal for the agent if $\theta = \frac{1}{2} - b$. When the preferences of the two agents are quite similar ($b \leq \frac{1}{7}$), the outside option becomes optimal for the agent for values of θ close to $\theta = \frac{1}{2}$. In such a case, for low and high states, the pressure induces by the outside option becomes detrimental for the agent, and the principal can capture some individual surplus by implementing a negative transfer.

On the contrary, when the preferences of the two agents are misaligned ($b \geq \frac{1}{4}$), the outside option becomes optimal for the agent for values of θ close to $\theta = 0$. In such a case, if the observed θ is low, the principal must implement a positive transfer to convince the agent to reveal information honestly. However, if the observed θ is high, a negative transfer can be implemented due to the outside option.

4.3 The sophisticated principal

In this section, we discuss the model by considering a sophisticated principal. Because we suppose that the principal knows the agent's bias perfectly, one can imagine that he could select an outside option based on this bias. We suppose that a sophisticated principal has such an ability. Does the use of the agent's bias in the determination of the outside option increase the principal's expected payoff ? Consider a contract with three separate pieces that satisfy the conditions (7) to (12) and where $y^H(\theta)$ is the optimal action taken on the subinterval $[0, \theta_1(k)]$, $\widetilde{y}$ the one on $[\theta_1(k), \theta_2(k)]$ and $y^L(\theta)$ the one on $[\theta_2(k), 1]$ with k a nonnegative parameter and $\theta_1(k) \neq \theta_2(k)$. By continuity of the decision rule $y(.)$, we construct the following feasible decision rule[4]

$$y(\theta) = \begin{cases} y^H(\theta) = \frac{3}{2}\theta + \frac{b}{2} \ \forall\theta \in [0, \frac{2k}{3}] \\ \widetilde{y} = k + \frac{b}{2} \ \forall\theta \in [\frac{2k}{3}, \frac{2k+1}{3}] \\ y^L(\theta) = \frac{3}{2}\theta + \frac{b-1}{2} \ \forall\theta \in [\frac{2k+1}{3}, 1] \end{cases}$$

where $\widetilde{y} = k + \frac{b}{2}$ denotes the outside option for the sophisticated principal. In order to guarantee a three separate pieces contract and that the transversality conditions are respected, we must have that $0 < \theta_1(k) < \theta_2(k) < 1$ that is equivalent to $k \in (0, 1)$.

The problem consists of determining a value of k that maximizes the principal's expected payoff. By continuity of the transfer, we have $t^H(\theta) = \frac{3}{4}\left[(\theta - b)^2 - (\frac{2k}{3} - b)^2\right]$ and $t^L(\theta) = \frac{3}{4}\left[(\theta - b - 1)^2 - (\frac{2(k-1)}{3} - b)^2\right]$. Thus, the principal's expected payoff ($EU^{\overline{P}}$) with the previous contract is

$$\begin{aligned} EU^{\overline{P}} &= \int_0^{\frac{2k}{3}} \left[-\left(\frac{\theta+b}{2}\right)^2 - \frac{3}{4}\left[(\theta-b)^2 - (\frac{2k}{3} - b)^2\right]\right] d\theta + \int_{\frac{2k}{3}}^{\frac{2k+1}{3}} -(k + \frac{b}{2} - \theta)^2 d\theta \\ &+ \int_{\frac{2k+1}{3}}^{1} \left[-\left(\frac{\theta+b-1}{2}\right)^2 - \frac{3}{4}\left[(\theta-b-1)^2 - (\frac{2(k-1)}{3} - b)^2\right]\right] d\theta \\ EU^{\overline{P}} &= \frac{1}{36}\left[4 + 18b - 9b^2 - 12k - 36bk + 12k^2\right] \end{aligned}$$

[4] The outside option $\widetilde{y} = k + \frac{b-1}{2}$ defined $\forall\theta[\frac{2k-1}{3}, \frac{2k}{3}]$ leads to the same results.

The expression of $EU^{\overline{P}}$ is decreasing with k and has a minimum for $k = \frac{1+3b}{2}$. Thus, since the naive principal's expected payoff EU^P is equal to $\frac{1}{3}b^2 + \frac{1}{36}$, a sufficient condition to have $EU^{\overline{P}} - EU^P > 0$ is given by $k \in \left(0, \frac{1-b}{2}\right)$. Thus, for values of the parameter $k \in \left(0, \frac{1-b}{2}\right)$, a sophisticated principal can generate a contract that strictly increases his expected payoff by comparison with the one of a naive principal. Because $EU^{\overline{P}}$ decreases with k, an optimal contract for a sophisticated principal consists of selecting $k = \varepsilon$ with $\varepsilon \to 0$. In the next example, we propose to exhibit a contract that is superior to the optimal contract for a naive principal.

Example 1 *Consider the case in which $k = \frac{b}{2}$ (and thus, $b < \frac{1}{2}$). The corresponding decision rule is*

$$y(\theta) = \begin{cases} y^H(\theta) = \frac{3}{2}\theta + \frac{b}{2} \ \forall \theta \in [0, \frac{b}{3}] \\ \widetilde{y} = b \ \forall \theta \in [\frac{b}{3}, \frac{b+1}{3}] \\ y^L(\theta) = \frac{3}{2}\theta + \frac{b-1}{2} \ \forall \theta \in [\frac{b+1}{3}, 1] \end{cases}$$

We also have $EU^{\overline{P}} = \frac{1}{36}\left[4 + 12b - 24b^2\right]$, and we easily verify that $EU^{\overline{P}} > EU^P$ for $b < \frac{1}{2}$.

In the previous contract, no payment is made for states $\theta \in [\frac{b}{3}, \frac{b+1}{3}]$, and the outside option $\widetilde{y} = b$ is taken.

4.4 Efficiency comparison

We conclude this section by comparing our optimal contracts with each other and with the one from Krishna and Morgan [2008]. We focus on the perfect commitment case of Krishna and Morgan [2008] with a positive constraint on the transfer and without an outside option. Contracts will be compared both from information and surplus extraction.

Information extraction We consider that information extraction is due to responsive mechanisms, that is, optimal mechanisms in which the principal chooses a different action for each state. First, note that when the principal can implement an outside option, his behavior does not affect information extraction. Indeed, when the principal is naive, the optimal contract implies information extraction for $\theta \in [0, \frac{1-b}{3}] \cup [\frac{2-b}{3}, 1]$, whereas with a sophisticated principal, information extraction arrives for $\theta \in [0, \frac{2k}{3}] \cup [\frac{2k+1}{3}, 1]$. Thus in a contract with an outside option, the principal is able to extract $\frac{2}{3}$ of the information held by the agent independently of the divergence of preferences.

The optimal contract characterized by Krishna and Morgan [2008] depends on the agent's bias b. For $b \leq \frac{1}{3}$, their optimal mechanism is responsive to the state for $\theta \in [0, 1-2b]$ whereas for $b > \frac{1}{3}$, it is the case for $\theta \in [0, a]$ with $a = \frac{1}{2} - \frac{1}{6}\sqrt{12b-3}$. Hence by comparing with their optimal contract, one can easily verify that for $b \geq \frac{1}{6}$ the use of the outside option increases the information extraction. Moreover for $b \geq \frac{1}{6}$, it is obvious to show that the higher the agent's bias is, the more the pressure induces by the outside option increases information revelation in our model.

Surplus extraction We propose to analyze surplus extraction for reasonable biases, that is, when $b \leq 1$. Indeed, Krishna and Morgan [2008] show that for very high biases, that is, when $b > 1$, contracting is of no use in their model.

Whatever the behavior of the principal (naive or sophisticated), there are two significative differences between our contract and the one of Krishna and Morgan [2008]. First, they show that for low bias ($b \leq \frac{1}{3}$), delegation is optimal on a non-degenerate subinterval of $[0, 1]$. Such a result is impossible in our model because we show that for low bias, the outside option induces the principal to implement a negative transfer. Second, in their model, pooling is optimal for high states. Again, in our model, for high states, the possibility to withhold the outside option allows the principal to implement a responsive decision rule and negative transfer. When the principal adopts a naive behavior, we can verify that the difference in the principal's expected payoff (Δ^{EU}) between our own contract (EU^P) and the one of Krishna and Morgan [2008] $\left(EU^P_{KM}\right)$ is given by

$$b \leq \frac{1}{3} : \Delta^{EU} = EU^P - EU^P_{KM} = \frac{1}{36}(1 + 48b^2 - 54b^3) > 0$$

$$b > \frac{1}{3} : \Delta^{EU} = EU^P - EU^P_{KM} = \frac{1}{72}\left[-1 + 42b^2 - 3\sqrt{12b-3} + 6b(3 + 2\sqrt{12b-3})\right] > 0.$$

Obviously, the previous computations indicate that the contract with an outside option is better for the principal regardless of the divergence of the preferences between the two agents. More precisely, when the bias increases, the difference in the expected payoff is in favor of the contract with an outside option. This result holds because we allow a transfer of surplus from the agent toward the principal.

Finally, we find that $\overline{EU^P} - EU^P > 0$. Thus, by comparing it with the optimal contract for a naive principal when the divergence of preferences between the principal and the agent are reasonable, the optimal contract for a sophisticated principal strictly improves the principal's expected payoff. In conclusion, from a surplus extraction point of view and in the case of uniform-quadratic loss utilities, our contract always dominates the one found by Krishna and Morgan [2008].

5 Conclusion

Starting from the classical results of the strategic information transmission literature, some papers find that the ability to contract for information reduces the loss of information and consequently improves decision-making. In a standard principal-agent setting, we complete these results by providing the opportunity for the principal to undertake an outside option and we analyze how the use of this outside option can favor the principal's interests.

Even if the principal has the opportunity to pay for information, we show that it is not always optimal to do so. In effect, we show that for some interior states, the principal prefers to implement his outside option; thus, no information is communicated by the agent. This loss of information is compensated by contracts that are responsive to the other states. For these contracts, the outside option can induce negative transfers that increase the surplus extraction from the agent to the principal. Finally, we compare our contract with the corresponding literature, and we show that *a)* our contract is always better with regard to surplus extraction, and *b)* information extraction is better in our model for $b \geq \frac{1}{6}$. A direct consequence of this result is that conflicting preferences can facilitate surplus and information extraction.

It remains for future research to study several extensions of the model. One interesting extension concerns the timing of the game. In our model, the principal offers a contract before the agent sends a message and we do not formalize the communication game. Because we consider a rather standard principal-agent model, the contract is optimally calculated such that the agent never refuses it. One can imagine different timing in which the principal would learn from the non-participation of the expert. This learning should have an important impact on the determination of the optimal contract.

Appendix 1: The general model

Proof of Proposition 1. Suppose that the contrary is true, that is, $y(\theta) = y^i(\theta)$ with $i \in \{A; P\}$ becomes implementable in an optimal contract. Recall that $y(\theta)$ must verify $\forall\theta \in [0,1]$: $\Phi_1(y^i(\theta), \theta, b)f(\theta) + \mu U_{12}(y^i(\theta), \theta, b) = 0$, and thus

$$\mu(\theta) = -\frac{\Phi_1(y^i(\theta), \theta, b)f(\theta)}{U_{12}(y^i(\theta), \theta, b)}$$

However, $\Phi_1(y^A(\theta), \theta, b) < 0$ (respectively, $\Phi_1(y^P(\theta), \theta, b) > 0$) implies that $\mu(\theta) > 0$ (respectively, $\mu(\theta) < 0$). For $\theta \in [0, \theta_1)$, $\frac{dU^A(\theta)}{d\theta} < 0$ and $U^A(\theta_1) = 0$ implies that $U^A(\theta) > 0$. We have $U^A(0) > 0$ and according to condition (5) $\mu(0) = 0$; therefore, there is a contradiction. For $\theta \in (\theta_2, 1]$, $\frac{dU^A(\theta)}{d\theta} > 0$, and $U^A(\theta_2) = 0$ implies that $U^A(\theta) > 0$. We have $U^A(1) > 0$ and according to condition (5) $\mu(1) = 0$, and there is again a contradiction. □

Proof of Lemma 1. We study three cases defined by the monotonicity and the continuity of the informational rent: (i) $\forall\theta \in [0, \theta_1]$, $y(\theta) \leq \widetilde{y} \Rightarrow \frac{dU^A(\theta)}{d\theta} \leq 0$, if $U^A(\theta_1) = 0$ then (IP) is satisfied and $U^A(\theta) \geq 0$. Moreover, $\forall\theta \in [0, \theta_1)$, $U^A(\theta) > 0$ and hence by (4) $\lambda(\theta) = 0$ and by (5) $\mu(0) = 0$. By integrating $\frac{d\mu}{d\theta}$ in condition (2) and by using $\mu(0) = 0$, we have that $\forall\theta \in [0, \theta_1]$, $\mu(\theta) = F(\theta)$.

(ii) $\forall\theta \in [\theta_1, \theta_2]$, $y(\theta) = \widetilde{y} \Rightarrow \frac{dU^A(\theta)}{d\theta} = 0$. To minimize the loss induced by the transfer, we have $\forall\theta \in [\theta_1, \theta_2]$ $U^A(\theta) = 0$ and $t(\theta) = 0$. Condition (4) implies that $\lambda(\theta) \geq 0$. By integrating $\frac{d\mu}{d\theta}$ into condition (2), we have $\forall\theta \in [\theta_1, \theta_2]$, $\mu(\theta) = F(\theta) - \int_{\theta_1}^{\theta} \lambda(s)\, ds + K$. By continuity of $\mu(\theta)$, we have $K = 0$.

(iii) $\forall\theta \in [\theta_2, 1]$, $y(\theta) \geq \widetilde{y} \Rightarrow \frac{dU^A(\theta)}{d\theta} > 0$, if $U^A(\theta_2) = 0$ then (IP) is satisfied and $U^A(\theta) \geq 0$. Moreover, $\forall\theta \in (\theta_2, 1]$, $U^A(\theta) > 0$ and hence by (4) $\lambda(\theta) = 0$ and by (5) $\mu(1) = 0$. By integrating $\frac{d\mu}{d\theta}$ in condition (2) and by using $\mu(1) = 0$, we have that $\forall\theta \in [\theta_2, 1]$, $\mu(\theta) = F(\theta) - 1$. □

Proof of Lemma 2. For $y(\theta) = \widetilde{y}$, condition (1) implies that

$$\mu(\theta) = -\frac{\Phi_1(\widetilde{y}, \theta, b)f(\theta)}{U_{12}(\widetilde{y}, \theta, b)}$$

By derivating $\mu(\theta)$, we obtain

$$\mu'(\theta) = -\left[\frac{[\Phi_{12}(\widetilde{y}, \theta, b)f(\theta) + \Phi_1(\widetilde{y}, \theta, b)f'(\theta)]\, U_{12}(\widetilde{y}, \theta, b) - U_{122}(\widetilde{y}, \theta, b)\Phi_1(\widetilde{y}, \theta, b)f(\theta)}{[U_{12}(\widetilde{y}, \theta, b)]^2}\right]$$

Thus, $\mu'(\theta) - f(\theta) =$

$$= -\left[\frac{\Phi_1(\widetilde{y}, \theta, b)\,[f'(\theta)\, U_{12}(\widetilde{y}, \theta, b) - U_{122}(\widetilde{y}, \theta, b)f(\theta)] + U_{12}(\widetilde{y}, \theta, b)f(\theta)\,[\Phi_{12}(\widetilde{y}, \theta, b) + U_{12}(\widetilde{y}, \theta, b)]}{[U_{12}(\widetilde{y}, \theta, b)]^2}\right]$$

A condition for $\mu'(\theta) - f(\theta) < 0$ is given by $\Phi_1(\widetilde{y}, \theta, b)\,[f'(\theta)\, U_{12}(\widetilde{y}, \theta, b) - U_{122}(\widetilde{y}, \theta, b)f(\theta)] \geq 0$. □

Proof of Proposition 2. 1) $i)$ Because $y^P(\theta) = \arg\max_y U(y, \theta)$ and $U_{13}(y, \theta, b) > 0 \Rightarrow$

$\Phi_1(y^P(\theta), \theta, b) > 0$. $\forall\theta \in [0, \theta_1]$ condition (1) implies that $\Phi_1(y^H(\theta), \theta, b) < 0$, and because $\Phi_{11}(y, \theta, b) < 0$, then $y^P(\theta) < y^H(\theta)$. *ii)* Incentive compatibility implies that t is nonincreasing if $U_1(y, \theta, b) > 0$. Because $y^A(\theta) = \arg\max_y U(y, \theta, b)$, we obtain $y^A(\theta) > y^H(\theta)$. 2) Because $y^A(\theta) = \arg\max_y U(y, \theta, b)$ and $U_{13}(y, \theta, b) > 0 \Rightarrow \Phi_1(y^A(\theta), \theta, b) < 0$. $\forall\theta \in [\theta_2, 1]$ condition (1) implies that $\Phi_1(y^L(\theta), \theta, b) > 0$, and because $\Phi_{11}(y, \theta, b) < 0$, then $y^L(\theta) < y^A(\theta)$ and $U_1(y^L(\theta), \theta, b) > 0$. 3) Condition (1) ensures the optimality of $y^*(\theta)$. Additionally, because $U_{12}(y, \theta, b) > 0$, condition (1) implies that for $\mu(\theta) = F(\theta)$, $\Phi_1(y^H(\theta), \theta, b) < 0$ and for $\mu(\theta) = F(\theta) - 1$, $\Phi_1(y^L(\theta), \theta, b) > 0$. Thus, because $\Phi_{11}(y, \theta, b) < 0$, we have $y^L(\theta) < y^H(\theta)$. □

Proof of Proposition 3. Incentive compatibility requires that $y(\theta)$ is nondecreasing and $t'(\theta) = -U_1(y, \theta, b)y'$. We know for $\theta \in [\theta_1, \theta_2]$ that $t(\theta) = 0$. Thus, by continuity, $t(\theta) < 0$ if *i)* $\forall\theta \in [0, \theta_1)$, $t'(\theta) > 0$ and *ii)* $\forall\theta \in (\theta_2, 1]$, $t'(\theta) < 0$.

i) $\forall\theta \in [0, \theta_1)$, by (IC) $t'(\theta) > 0$ if $U_1(y^H(\theta), \theta, b) > 0$. Proposition 2.1) implies that $\Phi_1(y^H(\theta), \theta, b) = U_1(y^H, \theta) + U_1(y^H, \theta, b) < 0$. Because $y^H(\theta) > y^P(\theta)$, $U_1(y^H, \theta) < 0$ and for some values of b, $U_1(y^H, \theta, b)$ can be positive. We know that $U_1(y^H, \theta, 0) < 0$ and $U_{13}(y, \theta, b) > 0$; thus, there exist a value $\underline{b}$ such that $U_1(y^H, \theta, \underline{b}) = 0$ and a value $\bar{b}$ such that $U_1(y^H, \theta) + U_1(y^H, \theta, \bar{b}) = 0$.

ii) $\forall\theta \in (\theta_2, 1]$, Proposition 2.3) implies that $U_1(y^L(\theta), \theta, b) > 0$, and so $t(\theta) < 0$. □

Appendix 2: The uniform-quadratic model

This appendix derives the solution in the uniform-quadratic case for the optimal contract with an outside option. In the uniform-quadratic case, the Pontryagin conditions (1) to (5) are also sufficient because the relevant convexity conditions are satisfied (Chiang, [1991]) and become:

$$\frac{\partial H}{\partial y} = 0 \Rightarrow y^* = \theta + \frac{b + \mu}{2} \tag{7}$$

Because $\Phi_{11}(y, \theta, b) < 0$, condition (7) is also sufficient for this maximization. The other sufficient conditions are

$$-\frac{\partial L}{\partial U^A} = \frac{d\mu}{d\theta} = 1 - \lambda \tag{8}$$

$$\begin{aligned} \frac{dU^A(\theta)}{d\theta} &= 2(y - \theta - b) - 2(\widetilde{y} - \theta - b) && (9) \\ &= 2(y - \widetilde{y}) && (10) \end{aligned}$$

$$\lambda(\theta) U^A(\theta) = 0, \ \lambda(\theta) \geq 0, \ U^A(\theta) \geq 0 \tag{11}$$

$$\mu(0) = 0 \text{ and } \mu(1) = 0 \tag{12}$$

A - The benchmark model

To solve the case in which the principal has no outside option, we must adapt the previous program by retaining $2(\widetilde{y}-\theta-b)=0$, which implies that $\frac{dU^A(\theta)}{d\theta}=2(y-\theta-b)=2(y-y^A)$. In such a case, we first establish a specific property of $y(\theta)$.

Lemma 3. There exists a $\widehat{\theta}<1$ such that for $[0,\widehat{\theta}]$, the action $y(\theta)$ is defined by $y(\theta)\leq y^A(\theta)$ and for $[\widehat{\theta},1]$ $y(\theta)=y^A(\theta)$.

Proof of Lemma 3. (IC) implies that $t^{'}(\theta)=2(y-y^A)y^{'}$. $t(\theta)=0$ only for $y(\theta)=y^A(\theta)$ (because $y=a$ never satisfies the individual rationality constraint). For $\theta\in[0,\widehat{\theta}]$, if $y(\theta)\leq y^A(\theta)$, $t^{'}(\theta)\leq 0$ and $\frac{dU^A(\theta)}{d\theta}\leq 0$. Moreover, for $\theta\in[\widehat{\theta},1]$ if $y(\theta)=y^A(\theta)$, $t(\theta)=0$ and $U^A(\theta)=0$. This satisfies that $U^A(\theta)\geq 0$.□

From Lemma 3, for all $\theta\in[0,1]$, $y(\theta)\leq y^A(\theta)$ and $\frac{dU^A(\theta)}{d\theta}\leq 0$. A direct consequence of Lemma 3 is that $U^A(1)=0$ and the state variable is not free at the extreme $\theta=1$, that is, condition (11) becomes $\mu(0)=0$ and $\mu(1)\neq 0$. The next proof establishes the design of the optimal contract in the case where the principal cannot implement an outside option.

Proof of proposition 4. Because $\frac{dU^A(\theta)}{d\theta}=2(y-y^A)$, we must consider two cases: *i)* $y<y^A\Rightarrow\frac{dU^A(\theta)}{d\theta}<0$. Suppose that there exists a value $\widehat{\theta}<1$ such that $U^A(\widehat{\theta})=0$; therefore, for all $\theta<\widehat{\theta}$, $U^A(\theta)>0$ implies that $\lambda(\theta)=0$. Condition (8) implies that $\mu(\theta)=\theta+c_1$, and because $\mu(0)=0$, we conclude that $\mu(\theta)=\theta$. Then, for all $\theta\in[0,\widehat{\theta}]$, $y(\theta)=\frac{3}{2}\theta+\frac{b}{2}$.
ii) $y=y^A\Rightarrow\frac{dU^A(\theta)}{d\theta}=0$. Suppose that it is verified on $[\widehat{\theta},1]$. Then, for all $\theta\in[\widehat{\theta},1]$ $U^A(\theta)=0$, and condition (11) implies that $\lambda(\theta)>0$. Condition (8) implies that $\mu(\theta)=\theta-\int_{\widehat{\theta}}^{\theta}\lambda(\theta)\,d\theta+c_2$ and by continuity $c_2=0$.
To conclude, by continuity of $y(\theta)$, we have that $\widehat{\theta}=b$ and for all $\theta\in[b,1]$, $\mu=b$ and $\lambda=1$.□

B - Outside option with the naive principal.

We summarize the solution in the following table:

	$0\leq\theta\leq\frac{1-b}{3}$	$\frac{1-b}{3}\leq\theta\leq\frac{2-b}{3}$	$\frac{2-b}{3}\leq\theta\leq 1$
$y(\theta)$	$\frac{3}{2}\theta+\frac{b}{2}$	$\frac{1}{2}$	$\frac{3}{2}\theta+\frac{b-1}{2}$
$t(\theta)$	$\frac{3}{4}\left[(\theta-b)^2-\frac{1}{9}(1-4b)^2\right]$	0	$\frac{3}{4}\left[(\theta-b-1)^2-\frac{1}{9}(1+4b)^2\right]$
$U^A(\theta)$	$\frac{1}{6}(3\theta+b-1)^2$	0	$\frac{1}{6}(3\theta+b-2)^2$
$\lambda(\theta)$	0	3	0
$\mu(\theta)$	θ	$1-2\theta-b$	$\theta-1$

Proof of Proposition 5. i) Notice that $t^H(\theta)$ is minimized for $a=\min\{b,\frac{1-b}{3}\}$ and $t^H(\theta)\big|_{\theta=\frac{1-b}{3}}=0$. Hence, we have that $t^H(\theta)\big|_{\theta=0}=(1-b)(7b-1)$. Thus, because $b<1$ if $b<\frac{1}{7}$, then $t^H(\theta)<0$ $\forall\theta\in[0,\frac{1-b}{3}]$. Moreover, one can easily verify that if $b\geq\frac{1}{4}$, $t^H(\theta)\geq 0$ $\forall\theta\in[0,\frac{1-b}{3}]$. ii) We have that $t'^L(\theta)\leq 0$ $\forall\theta\in[\frac{2-b}{3},1]$. Therefore, by continuity $t^L(\theta)\big|_{\theta=\frac{2-b}{3}}=0$, and $t^L(\theta) \leq 0$. □

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